Disturbance Decoupling and Unknown-Input Observation Problems

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Block 1: Foundations [10:30 - 12.30]:

- Talk 1: *Motivation and historical perspective*, **G. Marro** [10:30 - 11:00]
- Talk 2: *Invariant subspaces*, **L. Ntogramatzidis** [11:00 - 11:30]
- Talk 3: *Controlled invariance and invariant zeros*, **D. Prattichizzo** [11:30 - 12:00]
- Talk 4: *Conditioned invariance and state observation*, **F. Morbidi** [12:00 - 12:30]

Block 2: Problems and applications [15:30 - 17.30]:

- Talk 5: *Stabilization and self-bounded subspaces*, **L. Ntogramatzidis** [15:30 - 16:00]
- Talk 6: *Disturbance decoupling problems*, **L. Ntogramatzidis** [16:00 - 16:30]
- Talk 7: *LQR and H_2 control problems*, **D. Prattichizzo** [16:30 - 17:00]
- Talk 8: *Spectral factorization and H_2-model following*, **F. Morbidi** [17:00 - 17:30]

Outline

- **Disturbance decoupling problem (DDP)**
- Measurable Signal Decoupling Problem (MSDP)
- Previewed Signal Decoupling Problem (PSDP)

Problem Formulation

The disturbance decoupling problem by state feedback is the basic problem of the geometric approach. LTI system Σ ruled by

$$\Sigma : \quad \begin{cases} \dot{x}(t) = A\,x(t) + B\,u(t) + H\,h(t) & x(0) = x_0 \\ y(t) = C\,x(t), \end{cases}$$

where, for all $t \geq 0$,

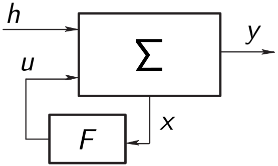
- $u(t) \in \mathbb{R}^m$ is the control input
- $h(t) \in \mathbb{R}^q$ is a disturbance to reject
- $y(t) \in \mathbb{R}^p$ is the output.

Determine $u(t) = F\,x(t)$ such that h has no influence on y and the overall system is internally stable.

Problem Formulation

Closed-loop system:

$$\begin{aligned} \dot{x}(t) &= (A + B\,F)\,x(t) + H\,h(t) \qquad x(0) = x_0 \\ y(t) &= C\,x(t) \end{aligned}$$



The DDP is equivalent to requiring that y converges to 0 as $t \to \infty$, or, equivalently, that

- the transfer function from h to y is zero
- $A + B\,F$ is stable $\Longrightarrow$ (A, B) must be stabilisable

Structural Condition

Ignore for now the stability requirement. We seek F such that $y = 0$ for any h with $x(0) = 0$. The output of

$$\begin{aligned} \dot{x}(t) &= (A + B\,F)\,x(t) + H\,h(t) \qquad x(0) = 0 \\ y(t) &= C\,x(t) \end{aligned}$$

is zero if and only if the reachable subspace is in $\ker C$. This is the smallest subspace which

- is $(A + B\,F)$-invariant
- contains $\mathcal{H} \stackrel{\text{def}}{=} \mathrm{im} H$

Since any $(A + BF)$-invariant subspace is (A, B)-controlled invariant, the problem has a solution if and only if

$$\boxed{\mathcal{H} \subseteq \mathcal{V}^{\star}_{(\mathcal{B},\mathcal{C})}}$$

Problem with stability: Solution 1

A simple solution to the problem with stability makes use of $\mathcal{V}_g^\star$:

$\mathcal{V}_g^\star$ is the largest output-nulling of (A, B, C) such that F exists such that

- $(A + B\,F)\,\mathcal{V}_g^\star \subseteq \mathcal{V}_g^\star \subseteq \ker\, C$
- $(A + B\,F)$ is stable

The DDP with stability has solutions if and only if

$$\boxed{\mathcal{H} \;\subseteq\; \mathcal{V}_g^\star}$$

How to compute $\mathcal{V}_g^\star$?

How to obtain $\mathcal{V}_g^\star$?

1. Compute $\mathcal{V}^\star_{(\mathcal{B},\mathcal{C})}$ and a friend F that assigns all its free poles;
2. Change basis using $T_1 = \left[\begin{array}{cc} T' & T'' \end{array}\right]$, where $\mathrm{im}\, T' = \mathcal{V}^\star_{(\mathcal{B},\mathcal{C})}$ to get

$$T_1^{-1}\,(A + B\,F)\,T_1 = \left[\begin{array}{cc} M_{SU} & \times \\ 0 & \times \end{array}\right],$$

   where $\sigma(M_{SU})$ are the internal poles of $\mathcal{V}^\star_{(\mathcal{B},\mathcal{C})}$.
3. A further basis change T_2 splits the stable from the antistable modes:

$$T_2^{-1}\,T_1^{-1}\,(A + B\,F)\,T_1\,T_2 \left[\begin{array}{ccc} M_S & 0 & \times \\ 0 & M_U & \times \\ 0 & 0 & \times \end{array}\right],$$

   The first columns of the product $T_1\,T_2$ span $\mathcal{V}_g^\star$.

Solution 1: $\mathcal{V}_g^\star$

The solvability condition

$$\operatorname{im} H \quad \subseteq \quad \mathcal{V}_g^\star$$

is concise and elegant.
Unfortunately:

- $\mathcal{V}_g^\star$ requires eigenspace computations;
- if we choose a friend of $\mathcal{V}_g^\star$ to implement the feedback, we are not exploiting all the freedom available in the assignment of the closed-loop poles.

Are there better solutions to this problem?

Solution 2: Self bounded subspaces

Consider

$$\Sigma : \quad \begin{cases} \dot{x}(t) = A\,x(t) + B\,u(t) + H\,h(t) & x(0) = x_0 \\ y(t) = C\,x(t) \end{cases}$$

Let us consider:

- the *undisturbed* system (A, B, C)
- the *disturbed* system $(A, [\,B \quad H\,], C)$

Let us consider the disturbed system $(A, [\,B \quad H\,], C)$, with $\mathcal{V}^\star_{(\mathcal{B}+\mathcal{H},\mathcal{C})}$ and $\mathcal{S}^\star_{(\mathcal{C},\mathcal{B}+\mathcal{H})}$, and

$$\Phi(\mathcal{B}+\mathcal{H},\mathcal{C}) = \{\mathcal{V} \in \mathcal{V}(A,\mathcal{B}+\mathcal{H},\mathcal{C}) \mid \mathcal{V} \supseteq \mathcal{V}^\star_{(\mathcal{B}+\mathcal{H},\mathcal{C})} \cap (\mathcal{B}+\mathcal{H})\}$$

Solution 2: Self bounded subspaces

It is very easy to prove that if

$$\mathcal{H} \subseteq \mathcal{V}^\star_{(\mathcal{B},\mathcal{C})}$$

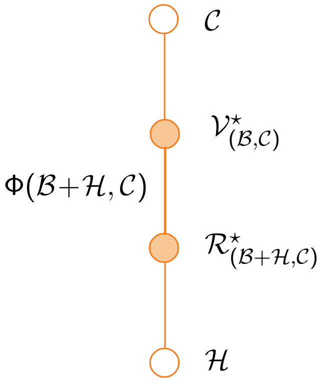
then

$$\mathcal{V}^\star_{(\mathcal{B}+\mathcal{H},\mathcal{C})} = \mathcal{V}^\star_{(\mathcal{B},\mathcal{C})}$$

Therefore,

$$\begin{aligned} \max \Phi(\mathcal{B}+\mathcal{H},\mathcal{C}) &= \mathcal{V}^\star_{(\mathcal{B},\mathcal{C})} \\ \min \Phi(\mathcal{B}+\mathcal{H},\mathcal{C}) &= \mathcal{R}^\star_{(\mathcal{B}+\mathcal{H},\mathcal{C})} = \mathcal{V}^\star_{(\mathcal{B},\mathcal{C})} \cap \mathcal{S}^\star_{(\mathcal{C},\mathcal{B}+\mathcal{H})} \end{aligned}$$

Solution 2: Self bounded subspaces



Lemma (Basile, Marro, Schumacher)

If $\mathcal{H} \subseteq \mathcal{V}^\star_{(\mathcal{B},\mathcal{C})}$, there exists at least one internally stabilizable output nulling $\mathcal{V}$ such that $\mathcal{H} \subseteq \mathcal{V} \subseteq \mathcal{C}$ if and only if $\mathcal{R}^\star_{(\mathcal{B}+\mathcal{H},\mathcal{C})}$ is internally stabilizable.

Solution of DDP with stability

DDP with stability admits a solution if and only if

$$\mathcal{H} \subseteq \mathcal{V}^\star_{(\mathcal{B},\mathcal{C})}$$

$$\mathcal{R}^\star_{(\mathcal{B}+\mathcal{H},\mathcal{C})} \quad \text{is internally stabilizable}$$

- $\mathcal{V}^\star_{(\mathcal{B},\mathcal{C})}$ is computed by

$$\begin{cases} \mathcal{V}_1 = \ker\ C \\ \mathcal{V}_i = \ker\ C \cap A^{-1}(\mathcal{V}_{i-1} + \mathrm{im}\, B) \quad i = 2, 3, \ldots, \end{cases}$$

- $\mathcal{R}^\star_{(\mathcal{B}+\mathcal{H},\mathcal{C})}$ is $\mathcal{V}^\star_{(\mathcal{B},\mathcal{C})} \cap \mathcal{S}^\star_{(\mathcal{C},\mathcal{B}+\mathcal{H})}$, where $\mathcal{S}^\star_{(\mathcal{C},\mathcal{B}+\mathcal{H})}$ is computed by

$$\begin{cases} \mathcal{S}_1 = \mathrm{im}\,[\,B \quad H\,] \\ \mathcal{S}_i = \mathrm{im}\,[\,B \quad H\,] + A(\mathcal{S}_{i-1} \cap \ker\ C) \quad i = 2, 3, \ldots, \end{cases}$$

Solution of DDP with stability

DDP with stability admits a solution if and only if

$$\mathcal{H} \subseteq \mathcal{V}^\star_{(\mathcal{B},\mathcal{C})}$$

$$\mathcal{R}^\star_{(\mathcal{B}+\mathcal{H},\mathcal{C})} \text{ is internally stabilizable}$$

Structural condition checked using MATLAB®:

```
>> V=vstar(A,B,C);
>> S=sstar(A,[B H],C);
>> R=ints(V,S);
>> if size(sums(H,V),2)==size(V,2),
     condition1='ok';
     else condition1='no';
     end
```

Solution of DDP with stability

DDP with stability admits a solution if and only if

$$\mathcal{H} \subseteq \mathcal{V}^\star_{(\mathcal{B},\mathcal{C})}$$

$$\mathcal{R}^\star_{(\mathcal{B}+\mathcal{H},\mathcal{C})} \text{ is internally stabilizable}$$

Stability condition checked using MATLAB®:

```
>> Rr=ints(R,miinco(A,R,B));
>> R1=ima([Rr R],0);
>> L=pinv([R1 B])*A*R1;
>> r=size(R,2); q=size(Rr,2);
>> X=L(q+1:r,q+1:r);
>> eig(X),
```

Solution of DDP with stability

DDP with stability admits a solution if and only if

$$\mathcal{H} \subseteq \mathcal{V}^\star_{(\mathcal{B},\mathcal{C})}$$

$$\mathcal{R}^\star_{(\mathcal{B}+\mathcal{H},\mathcal{C})} \quad \text{is internally stabilizable}$$

Construction of F using MATLAB®:

```
>> F=effesta(A,B,R);
```

Solution of DDP with stability

DDP with stability admits a solution if and only if

$$\mathcal{H} \subseteq \mathcal{V}^\star_{(\mathcal{B},\mathcal{C})}$$

$$\mathcal{R}^\star_{(\mathcal{B}+\mathcal{H},\mathcal{C})} \text{ is internally stabilizable}$$

- Separation between *structural* and *stability* conditions;
- These conditions are constructive: the decoupling F is an internally stabilizing friend of $\mathcal{R}^\star_{(\mathcal{B}+\mathcal{H},\mathcal{C})}$;
- The stability condition is satisfied if the zeros are all stable since $\mathcal{Z}(\mathcal{R}^\star_{(\mathcal{B}+\mathcal{H},\mathcal{C})}) \subseteq \mathcal{Z}(\mathcal{V}^\star_{(\mathcal{B},\mathcal{C})})$
- $\mathcal{R}^\star_{(\mathcal{B}+\mathcal{H},\mathcal{C})} \subseteq \mathcal{V}^\star_g$
- $\mathcal{R}^\star_{(\mathcal{B}+\mathcal{H},\mathcal{C})}$ ensures maximum freedom in assigning the closed-loop modes

Example 1

Consider the continuous-time system

$$A = \begin{bmatrix} -50 & 0 & 0 & -20 \\ 0 & 10 & 0 & 80 \\ 20 & 0 & -70 & 0 \\ 10 & 0 & 0 & -100 \end{bmatrix}, \; B = \begin{bmatrix} 0 & 0 \\ 40 & 0 \\ 30 & -100 \\ 0 & 0 \end{bmatrix}, \; H = \begin{bmatrix} 2 \\ 0 \\ 1 \\ 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & -5 & 0 & 0 \end{bmatrix}.$$

We compute

$$\mathcal{V}^\star_{(\mathcal{B},\mathcal{C})} = \mathrm{im} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathcal{S}^\star_{(\mathcal{C},\mathcal{B}+\mathcal{H})} = \mathbb{R}^4 \quad \Longrightarrow \quad \mathcal{R}^\star_{(\mathcal{B}+\mathcal{H},\mathcal{C})} = \mathcal{V}^\star_{(\mathcal{B},\mathcal{C})}.$$

Hence, $\mathrm{im}\, H \subseteq \mathcal{V}^\star_{(\mathcal{B},\mathcal{C})}$. Moreover,
$\sigma(X) = \{-75 + 5\sqrt{17},\, -75 - 5\sqrt{17}\} \subset \mathbb{C}_g$.

Example 1

Using `effesta.m` we compute

$$F = \begin{bmatrix} 0 & -3/10 & 0 & -2 \\ 1/5 & 0 & -69/100 & -3/5 \end{bmatrix}$$

- 1 free internal eigenvalue: -1
- 1 free external eigenvalue: -2

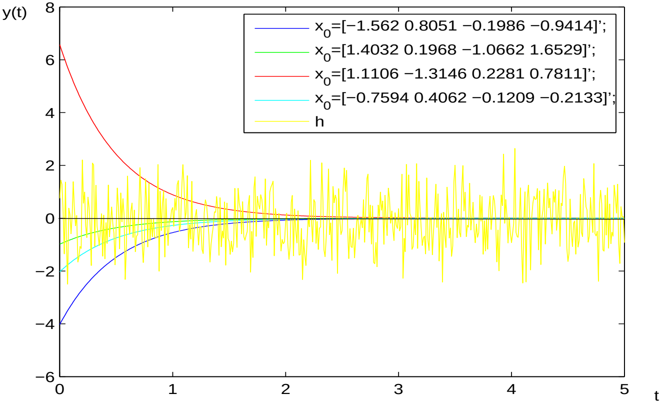
$$A+B\,F = \begin{bmatrix} -50 & 0 & 0 & -20 \\ 0 & -2 & 0 & 0 \\ \frac{1}{500} & -9 & -1 & -\frac{3}{500} \\ 10 & 0 & 0 & -100 \end{bmatrix} \Longrightarrow \quad \sigma(A+B\,F) = \left\{ \begin{array}{c} -1 \\ -2 \\ -75+5\sqrt{17} \\ -75-5\sqrt{17} \end{array} \right\}.$$

In this case

$$\sigma_{\text{fixed}}^{\text{int}}(\mathcal{R}^{\star}_{(\mathcal{B}+\mathcal{H},\mathcal{C})}) = \mathcal{Z}(A,B,C).$$

Example 1

Time response for different initial condition



Example 2

Consider

$$A = \begin{bmatrix} -2 & 0 & 0 & 0 \\ 0 & -8 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \; B = \begin{bmatrix} 0 & 0 \\ 0.5 & -0.2 \\ 0 & 0 \\ 0.7 & 0 \end{bmatrix}, \; H = \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & 0 & -10 & -40 \end{bmatrix}.$$

Now

$$\mathcal{V}^\star_{(\mathcal{B},\mathcal{C})} = \mathrm{im} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1/4 \end{bmatrix}, \quad \mathcal{S}^\star_{(\mathcal{C},\mathcal{B}+\mathcal{H})} = \mathrm{im} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Longrightarrow \mathcal{R}^\star_{(\mathcal{B}+\mathcal{H},\mathcal{C})} = \mathrm{im} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

Example 2

Now, the zeros are:

$$\mathcal{Z}(A,B,C) = \{-2,\ \frac{5}{2}\}$$

$\Longrightarrow$ non-minimum phase system!

However, $\mathcal{R}^{\star}_{(\mathcal{B}+\mathcal{H},\mathcal{C})}$ has

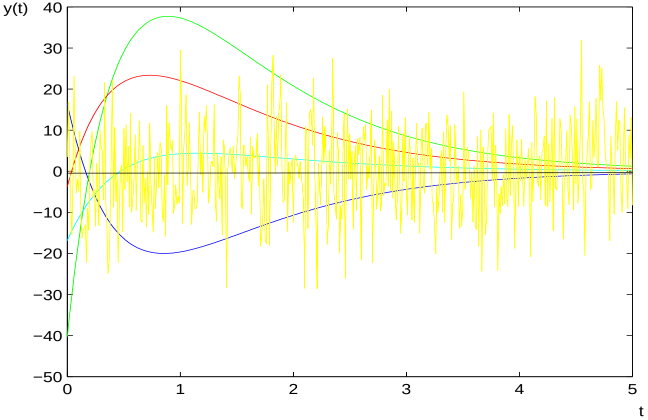
- 1 fixed internal eigevalue -2
- 1 free internal eigenvalue; we choose -1
- 2 free external eigenvalues; we choose -3 and -4

$\Longrightarrow$ $\mathcal{R}^{\star}_{(\mathcal{B}+\mathcal{H},\mathcal{C})}$ is internally stabilizable

$$F = \begin{bmatrix} 0 & 0 & 12/7 & -10 \\ 0 & -35 & 0 & 0 \end{bmatrix} \Longrightarrow \sigma(A+B\,F) = \left\{ \begin{array}{c} -1 \\ -2 \\ -3 \\ -4 \end{array} \right\}$$

Example 2

Time response for different initial condition



Self bounded subspaces and fixed poles

- $\mathcal{R}^\star_{(\mathcal{B}+\mathcal{H},\mathcal{C})}$ is not the smallest internally stabilizing subspace that contains $\mathcal{H}$
- However, choosing $\mathcal{R}^\star_{(\mathcal{B}+\mathcal{H},\mathcal{C})}$ to construct F is the best solution for the assignment of the closed-loop dynamics
- When DDP is solvable, a maximal set of eigenvalues of the closed-loop exists which is present for any feedback solution $\longrightarrow$ fixed poles of the decoupling problem
- At least one F exists such that all the remaining eigenvalues can be assigned arbitrarily

Theorem: the fixed poles of the DDP are the unassignable eigenvalues of $\mathcal{R}^\star_{(\mathcal{B}+\mathcal{H},\mathcal{C})}$, i.e.,

- $\sigma_{\rm fixed}(\mathcal{R}^\star_{(\mathcal{B}+\mathcal{H},\mathcal{C})}) \subseteq \sigma(A+B\,F)$ for any F that solves the DDP;
- There is no other fixed eigenvalue present in all solutions.

Outline

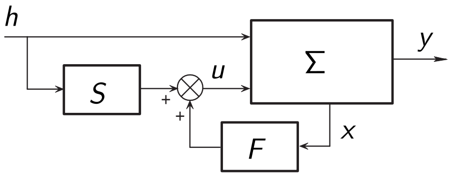
- Disturbance decoupling problem (DDP)
- **Measurable Signal Decoupling Problem (MSDP)**
- Previewed Signal Decoupling Problem (PSDP)

Problem Formulation

The MSDP by static feedback/feedforward can be stated as follows

$$\Sigma: \quad \begin{cases} \dot{x}(t) = A\,x(t) + B\,u(t) + H\,h(t), & x(0) = x_0, \\ y(t) = C\,x(t). \end{cases}$$

Determine, if possible, a control law $u(t) = F\,x(t) + S\,h(t)$ such that h has no influence on output y and the overall system is internally stable.



Structural Condition

The closed-loop system is

$$\Sigma : \quad \begin{cases} \dot{x}(t) = (A + B\,F)\,x(t) + (H + B\,S)\,h(t) \qquad x(0) = x_0 \\ y(t) = C\,x(t), \end{cases}$$

Using the new d.o.f. S, we can instantaneously reject the components of h on $\operatorname{im} B$.

Structural condition:

$$\boxed{\mathcal{H} \;\subseteq\; \mathcal{V}^\star_{(\mathcal{B},\mathcal{C})} + \mathcal{B}}$$

$\Longrightarrow$ H can be written as $H = V\,\Pi_1 + B\,\Pi_2$, where $\operatorname{im} V = \mathcal{V}^\star_{(\mathcal{B},\mathcal{C})}$.

Structural Condition

The closed-loop system is

$$\Sigma : \quad \begin{cases} \dot{x}(t) = (A + B\,F)\,x(t) + V\,\Pi_1\,h(t) + B\,(\Pi_2 + S)\,h(t) \\ y(t) = C\,x(t), \end{cases}$$

Solution: $S = -\Pi_2$, and F is a friend of $\mathcal{V}^\star_{(\mathcal{B},\mathcal{C})}$.

Solution 1 with stability:

$$\boxed{\mathcal{H} \subseteq \mathcal{V}^\star_g + \mathcal{B}}$$

Complete Solution

Lemma (Basile, Marro, Schumacher)

If $\mathcal{H} \subseteq \mathcal{V}^{\star}_{(\mathcal{B},\mathcal{C})} + \mathrm{im}\, B$, there exists at least one internally stabilizable output nulling $\mathcal{V}$ such that $\mathcal{H} \subseteq \mathcal{V} \subseteq \mathcal{C}$ if and only if $\mathcal{R}^{\star}_{(\mathcal{B}+\mathcal{H},\mathcal{C})}$ is internally stabilizable.

The solvability conditions are:

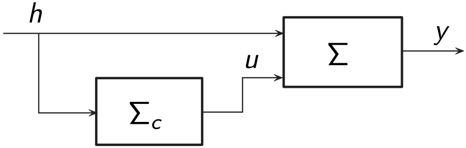
$$\mathcal{H} \subseteq \mathcal{V}^{\star}_{(\mathcal{B},\mathcal{C})} + \mathcal{B}$$

$$\mathcal{R}^{\star}_{(\mathcal{B}+\mathcal{H},\mathcal{C})} \;\; \text{is internally stabilizable}$$

- the stability condition does not change
- this solution still guarantees maximum freedom in the assignment of closed-loop poles

Open loop solution of MSDP

If Σ is stable, we can also formulate the MSDP using a purely feedforward dynamic compensator Σ_c:



The solvability conditions do not change. Let R be a basis of $\mathcal{R}^\star_{(\mathcal{B}+\mathcal{H},\mathcal{C})}$.

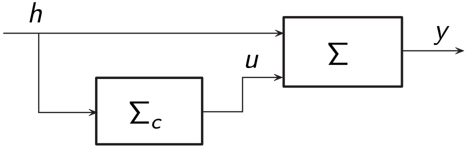
- $\mathcal{H} \subseteq \mathcal{V}^\star_{(\mathcal{B},\mathcal{C})} + \mathcal{B} \implies \mathcal{H} \subseteq \mathcal{R}^\star_{(\mathcal{B}+\mathcal{H},\mathcal{C})} + \mathcal{B} \implies H = R\,\Pi_1 + B\,\Pi_2$;
- there exists X such that $(A + B\,F)\,R = R\,X$.

The decoupling compensator is

$$\boxed{\Sigma_c = (X, \Pi_1, F\,R, -\Pi_2)}$$

Open loop solution of MSDP

If we use $\Sigma_c = (X, \Pi_1, F\,R, -\Pi_2)$:



$$\begin{aligned} \dot{x} &= A\,x + B\,u + H\,h \\ y &= C\,x \\ \dot{z} &= X\,z + \Pi_1\,h \\ u &= F\,R\,z - \Pi_2\,h \end{aligned} \quad \Longrightarrow \quad \begin{aligned} \dot{x} &= A\,x + B\,F\,R\,z \\ &\quad -\cancel{B\,\Pi_2\,h} + R\,\Pi_1\,h \\ &\quad +\cancel{B\,\Pi_2\,h} \\ y &= C\,x \end{aligned}$$

Open loop solution of MSDP

If we use $\Sigma_c = (X, \Pi_1, F\,R, -\Pi_2)$:

$$\begin{aligned} \dot{x} &= A\,x + B\,F\,R\,z + R\,\Pi_1\,h \\ y &= C\,x \end{aligned}$$

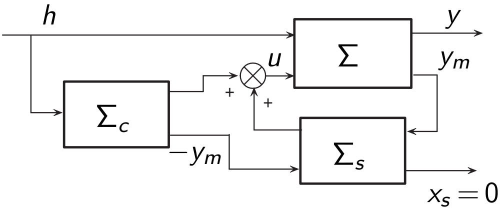
behaves like

$$\begin{aligned} \dot{x} &= (A + B\,F)\,x + R\,\Pi_1\,h \\ y &= C\,x \end{aligned}$$

if $x(0) = 0$ and $z(0) = 0$, since $x = R\,z$.

- the state of Σ_c coincides with the coordinates of x over $\mathcal{R}^\star_{(\mathcal{B}+\mathcal{H},\mathcal{C})}$
- the order of Σ_c is the dimension of $\mathcal{R}^\star_{(\mathcal{B}+\mathcal{H},\mathcal{C})}$

Open loop solution of MSDP



If Σ is stabilizable from u and detectable from an output y_m (possibly coinciding with y), it can be stabilized with Σ_s, which can be maintained at zero by Σ_c since this reproduces the state evolution (hence the output) of Σ, restricted to $\mathcal{R}^\star_{(\mathcal{B}+\mathcal{H},\mathcal{C})}$. Let F_s be such that $A + B\,F_s$ is stable and G_s such that $A + G_s\,C_1$ is strictly stable.

$$\Sigma_s \;=\; (A + BF_s + G_s C_1\,,\, -G_s\,,\, F_s).$$

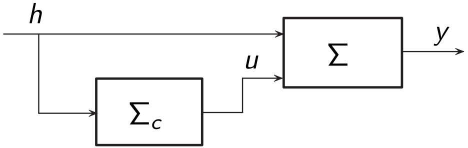
The output of Σ_s is zero, since its inputs due to the action of h on Σ and Σ_c cancel each other.

Dualizing MSDP

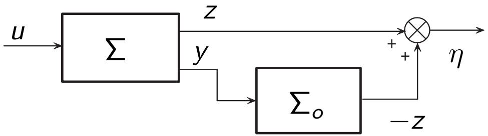
Recall that

- the dual of a summing junction is a branching point
- the dual of (A, B, C, D) is $(A^{\top}, C^{\top}, B^{\top}, D^{\top})$

Hence, the dual of



is

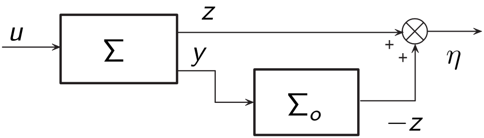


Unknown-Input Observer

Consider a system with two outputs:

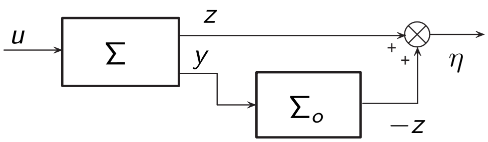
$$\begin{aligned} \dot{x} &= A\,x + B\,u \\ z &= E\,x \\ y &= C\,x \end{aligned}$$

Determine, if possible, an observer Σ_o such that the input u has no influence on output η:



Σ_o is an unknown-input observer. This problem can be solved by duality.

Unknown-Input Observer: Solution



By dualizing

$$\mathcal{H} \subseteq \mathcal{V}^\star_{(\mathcal{B},\mathcal{C})} + \mathcal{B}$$
$$\mathcal{R}^\star_{(\mathcal{B}+\mathcal{H},\mathcal{C})} \quad \text{is internally stabilizable}$$

we get

$$\mathcal{S}^\star_{(\mathcal{C},\mathcal{B})} \cap \mathcal{C} \subseteq \mathcal{E}$$
$$\mathcal{S}^\star_{(\mathcal{C},\mathcal{B})} + \mathcal{V}^\star_{(\mathcal{B},\mathcal{C}\cap\mathcal{E})} \quad \text{is externally stabilizable}$$

Outline

- Disturbance decoupling problem (DDP)
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- **Previewed Signal Decoupling Problem (PSDP)**

Previewed Signal Decoupling Problem

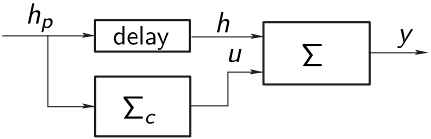
Consider the discrete-time system

$$\Sigma : \quad \begin{cases} x(k+1) = A\,x(k) + B\,u(k) + H\,h(k), & x(0) = x_0, \\ y(k) = C\,x(k). \end{cases}$$

Suppose h is previewed by q instants. Define

$$h_p(k) = h(k+q)$$

Find Σ_c such that h_p has no influence on y.



Previewed Signal Decoupling Problem: Intuitive Idea

Suppose $x(0) = 0$ and

$$h_p(0) = \bar{h} \qquad \Longrightarrow \qquad h(q) = \bar{h}$$

Hence,

$$\begin{aligned} x(1) &= B\,u(0) \\ &\vdots \\ x(q) &= \sum_{i=0}^{q-1} A^{q-i-1}\,B\,u(i) \\ x(q+1) &= \sum_{i=0}^{q} A^{q-i-1}\,B\,u(i) + H\,\bar{h} \end{aligned}$$

Then $u(t)$ can reject:

- the components of h on $\mathcal{V}^\star_{(\mathcal{B},\mathcal{C})}$
- the components of h on $\mathrm{im}\,B$
- the components of h on another subspace $\mathcal{L}$

Previewed Signal Decoupling Problem: Intuitive Idea

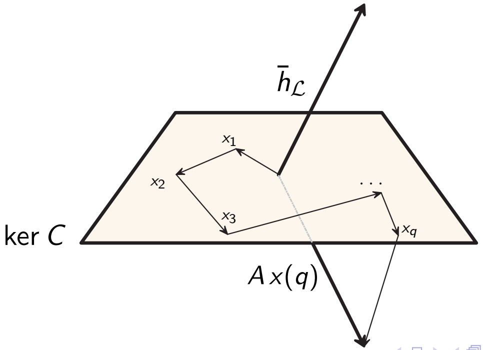
As such,

$$H\,\bar{h} = \bar{h}_{\mathcal{V}^\star} + \bar{h}_{\mathcal{B}} + \bar{h}_{\mathcal{L}},$$

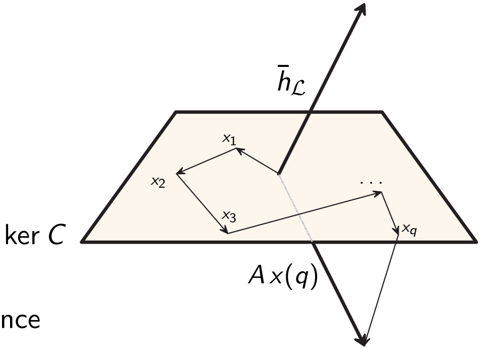
so that

$$x(q+1) = A\,x(q) + \cancel{B\,u(q)} + \bar{h}_{\mathcal{V}^\star} + \cancel{\bar{h}_{\mathcal{B}}} + \bar{h}_{\mathcal{L}}$$

$\Longrightarrow$ we need $\bar{h}_{\mathcal{L}} = -A\,x(q)$:



Previewed Signal Decoupling Problem: Intuitive Idea



Recall the sequence

$$\mathcal{S}_1 = \operatorname{im} B$$
$$\mathcal{S}_i = \operatorname{im} B + A\,(\mathcal{S}_{i-1} \cap \ker C)$$

The states on $\mathcal{S}_k$ are reachable from $\{0\}$ in k steps, with outputs $y(1) = y(2) = \ldots = y(k-1) = 0 \implies \mathcal{L} = \mathcal{S}_q$.

Previewed Signal Decoupling Problem: Solution

The solvability conditions are:

$$\mathcal{H} \subseteq \mathcal{V}^\star_{(\mathcal{B},\mathcal{C})} + \mathcal{S}_q$$
$$\mathcal{R}^\star_{(\mathcal{B}+\mathcal{H},\mathcal{C})} \text{ is internally stabilizable}$$

where

$$\begin{aligned} \mathcal{S}_1 &= \operatorname{im} B \\ \mathcal{S}_i &= \operatorname{im} B + A\,(\mathcal{S}_{i-1} \cap \ker\, C) \end{aligned}$$

- The stabilizability condition does not change
- The structural condition is more likely to be satisfied if q increases, until the sequence stops increasing

Previewed Signal Decoupling Problem: Solution

If q is not a prescribed integer but a design parameter, the solvability conditions become

$$\mathcal{H} \subseteq \mathcal{V}^{\star}_{(\mathcal{B},\mathcal{C})} + \mathcal{S}^{\star}_{(\mathcal{C},\mathcal{B})}$$
$$\mathcal{R}^{\star}_{(\mathcal{B}+\mathcal{H},\mathcal{C})} \quad \text{is internally stabilizable}$$

- The structural condition is satisfied if Σ is right invertible
- The stabilizability condition is satisfied if Σ is minimum phase

Dual: