Motivations and Historical Perspective

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MTNS - July 5-9, 2010

Block 1: Foundations [10:30 - 12.30]:

- Talk 1: *Motivation and historical perspective*, **G. Marro** [10:30 - 11:00]
- Talk 2: *Invariant subspaces*, **L. Ntogramatzidis** [11:00 - 11:30]
- Talk 3: *Controlled invariance and invariant zeros*, **D. Prattichizzo** [11:30 - 12:00]
- Talk 4: *Conditioned invariance and state observation*, **F. Morbidi** [12:00 - 12:30]

Block 2: Problems and applications [15:30 - 17.30]:

- Talk 5: *Stabilization and self-bounded subspaces*, **L. Ntogramatzidis** [15:30 - 16:00]
- Talk 6: *Disturbance decoupling problems*, **L. Ntogramatzidis** [16:00 - 16:30]
- Talk 7: *LQR and H_2 control problems*, **D. Prattichizzo** [16:30 - 17:00]
- Talk 8: *Spectral factorization and H_2-model following*, **F. Morbidi** [17:00 - 17:30]

What the geometric approach is

The geometric approach is a collection of tools, properties and algorithms for the analysis and control of dynamic linear and nonlinear systems in a coordinate-free context.

Statements in terms of subspaces (or, more generally, sets) instead of matrices are compact and clean, and insight into their meaning is highly facilitated.

This mini-course is an attempt to present in simple terms the origin, motivation and growth of the geometric approach through almost four decades.

Outline

- **The beginning**
- The main tools
- The most important solved problems

Our first papers in English

Controlled and Conditioned Invariant Subspaces in Linear System Theory[1]

G. Basile[2] and G. Marro[3]

Communicated by G. Leitmann

Abstract. The concept of invariance of a subspace under a linear transformation is strongly connected with controllability and observability of linear dynamical systems. In this paper, we define *controlled* and *conditioned* invariant subspaces as a generalization of the simple invariants, for the purpose of investigating some further structural properties of linear systems. Moreover, we prove some fundamental theorems on which the computation of the above-mentioned subspaces is based. Then, we give two examples of practical application of the previous concepts concerning the determination of the constant output and perfect output controllability subspaces.

On the Observability of Linear, Time-Invariant Systems with Unknown Inputs[1]

G. Basile[2] and G. Marro[3]

Communicated by G. Leitmann

Our first papers in English

JOTA: VOL. 3, NO. 5, 1969 315

References

1. Kalman, R. E., *On the General Theory of Control Systems*, Proceedings of the First IFAC Congress, Vol. 1, Butterworths, London, 1961.
2. Kalman, R. E., *Canonical Structure of Linear Dynamical Systems*, Proceedings of National Academy of Sciences, Vol. 48, 1962.
3. Kalman, R. E., Ho, Y. C., and Narendra, K. S., *Controllability of Linear Dynamical Systems*, Contributions to Differential Equations, Vol. 1, 1962.
4. Gilbert, E. G., *Controllability and Observability in Multivariable Control Systems*, SIAM Journal on Control, Vol. 1, No. 2, 1963.
5. Kalman, R. E., *Mathematical Description of Linear Dynamical Systems*, SIAM Journal on Control, Vol. 1, No. 2, 1963.
6. Kreindler, E., and Sarachik, P. E., *On the Concepts of Controllability and Observability in Linear Systems*, IEEE Transactions on Automatic Control, Vol. AC-9, No. 2, 1964.
7. Weiss, L., Kalman, R. E., *Contributions to Linear System Theory*, International Journal of Engineering Science, Vol. 3, No. 2, 1965.
8. Silverman, L. M., and Meadows, H. E., *Controllability and Observability in Time-Variable Linear Systems*, SIAM Journal on Control, Vol. 5, No. 1, 1967.
9. Basile, G., Laschi, R., and Marro, G., *Luoghi Caratteristici delle Traiettorie dei Sistemi Lineari nello Spazio delle Uscite*, L'Elettrotecnica, Vol. 56, No. 5, 1969.
10. Basile, G., and Marro, G., *On the Observability of Linear Time-Invariant Systems with Unknown Inputs*, Journal of Optimization Theory and Applications, Vol. 3, No. 6, 1969.

Independently

- The following words can often be found in the literature: *"the geometric approach was introduced independently by Basile and Marro and Wonham and Morse"*. This is true. The following papers are usually cited:

G. Basile and G. Marro,
"Controlled and conditioned invariant subspaces in linear system theory,"
J. Optim. Theory Appl., vol. 3, no. 5, pp. 306–315, 1969.

W. M. Wonham and A. S. Morse,
"Decoupling and pole assignment in linear multivariable systems: a geometric approach,"
SIAM J. Contr., vol. 8, no. 1, pp. 1–18, 1970.

- Let us look at two interesting documents concerning that period.

The end of the period of independence (January 29, 1970)

University of Toronto

TORONTO 5. CANADA

DEPARTMENT OF ELECTRICAL ENGINEERING

January 29, 1970

Prof. G. Marro
Computing & Servomechanism Center
University of Bologna
Bologna, Italy

Dear Professor Marro:

Recently Prof. M. Aoki brought to my attention your interesting recent papers in J. Opt. Th. and Appl., 3(5) and 3(6), 1969. In these papers you have exploited a geometric approach which, independently, A.S. Morse and I have also found to be very powerful in certain problems of multivariable linear synthesis.

Our recent reports are enclosed for your interest, and I would, of course, greatly appreciate receiving any further material in this area which you may have available for distribution.

With best wishes,

Sincerely yours,

W. M. Wonham
Associate Professor

WMW:jdl
Encl:

Another significant letter received in that period

UNIVERSITY OF SOUTHERN CALIFORNIA
UNIVERSITY PARK LOS ANGELES, CALIFORNIA 90007

SCHOOL OF ENGINEERING
DEPARTMENT OF ELECTRICAL ENGINEERING

Dear Dr Marro

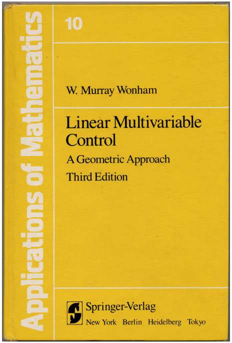
I have noted with interest that you and Dr Basile have been pursuing a theory of linear systems closely related to that being developed by my colleagues and myself here (as well as Wonham and Morse). I would appreciate receiving reprints of your work in the area and am enclosing several of my own. I am particularly interested at the moment in various types of observability (with differentiators and observers) which do not require input knowledge.

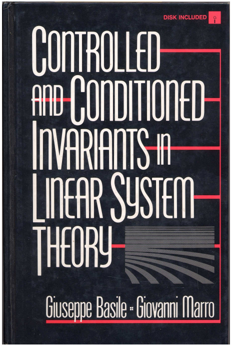
I am looking forward to hearing from you.

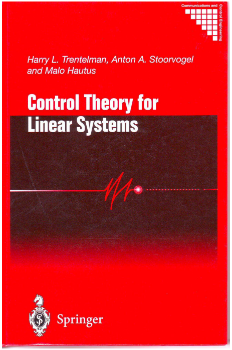
Sincerely yours

Leonard Silverman

The three books on the geometric approach







Wonham (1974, 1979, 1985) has the great merit of having widely publicized the geometric approach.

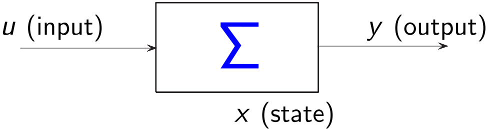
Basile and Marro (1992) emphasize duality and dual problems, that were neglected in Wonham's book. Software for Matlab is included.

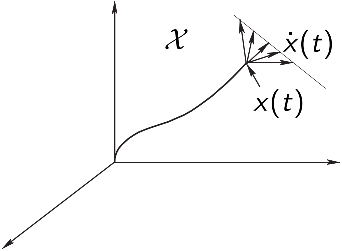
Trentelman, Stoorvogel and Hautus (2001) provide a bridge towards achieving minimal H_2 and H_∞ norm solutions.

Outline

- The beginning
- **The main tools**
- The most important solved problems

The concept of trajectory in the state space





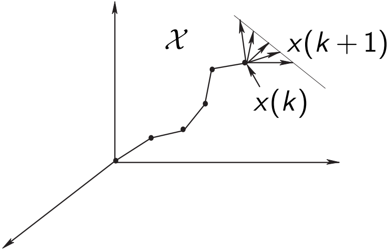
Continuous-time:

$$\begin{aligned} \dot{x}(t) &= A\,x(t) + B\,u(t) \\ y(t) &= C\,x(t) \end{aligned}$$

$$\begin{aligned} \dot{x}(t) &= A\,x(t) + B\,u(t) \\ y(t) &= C\,x(t) + D\,u(t) \end{aligned}$$

the term $D\,u(t)$
is called *feedthrough*.

The concept of trajectory in the state space



Discrete-time:

$$\begin{aligned} x(k+1) &= A\,x(k) + B\,u(k) \\ y(k) &= C\,x(k) \end{aligned}$$

$$\begin{aligned} x(k+1) &= A\,x(k) + B\,u(k) \\ y(k) &= C\,x(k) + D\,u(k) \end{aligned}$$

Very often in practice the discrete-time systems are produced by sampling continuous-time systems, for instance with ZOH (zero order hold) equivalence. In these case a positive *sampling time* is associated to the conversion process.

The Matlab Representations of LTI Systems

Purely dynamic continuous-time systems

```
>> sys=ss(A,B,C,0);
```

Continuous-time systems with feedthrough

```
>> sys=ss(A,B,C,D);
```

Purely dynamic discrete-time systems

`>> sys1=ss(A,B,C,0,Tc);` ($Tc = -1$ if a sampling time is not specified)

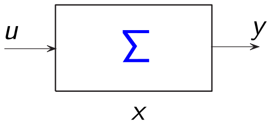
Discrete-time systems with feedthrough

```
>> sys1=ss(A,B,C,D,Tc);
```

Conversion from continuous to discrete time with the ZOH equivalence

`>> sys1=c2d(sys,Tc);` ($Tc > 0$ - in this case a sampling time is specified)

Some properties of systems expressed in geometric terms

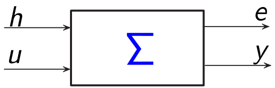



```
>> V=vstar(sys);
>> S=sstar(sys);
>> r=reldeg(sys);
>> z=gazero(sys);
```

- Left invertibility $\mathcal{V}^* \cap \mathcal{S}^* = \{0\}$
- Right invertibility $\mathcal{V}^* + \mathcal{S}^* = \mathcal{X}$
- Relative degree minimal ρ such that $\mathcal{V}^* + \mathcal{S}_\rho = \mathcal{X}$
- Minimality of phase $\mathcal{Z}(\Sigma) \in \mathbb{C}_g$

  where $\mathbb{C}_g$ is the set of stable modes in the complex plane.

Systems with multiple sets of inputs and outputs



h : disturbance
e : controlled output
u : control input
y : measured output

The system equation are

$$\begin{aligned} \dot{x}(t) &= A\,x(t) + H\,h(t) + B\,u(t) \\ e(t) &= E\,x(t) \qquad\qquad\qquad + D_1\,u(t) \\ y(t) &= C\,x(t) + D_2\,h(t) \end{aligned}$$

or

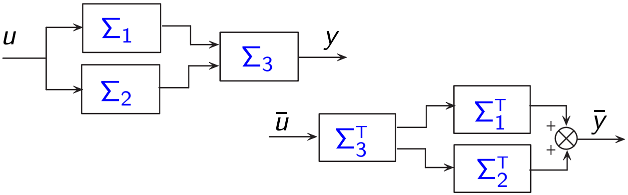
$$\begin{bmatrix} \dot{x}(t) \\ e(t) \\ y(t) \end{bmatrix} = \left[\begin{array}{c|cc} A & H & B \\ \hline E & 0 & D_1 \\ C & D_2 & 0 \end{array}\right] \begin{bmatrix} x(t) \\ h(t) \\ u(t) \end{bmatrix}$$

or else, by using a popular notation that points out the existence of feedthroughs terms, we can denote the system with

$$\Sigma = \left[\begin{array}{c|cc} A & H & B \\ \hline E & 0 & D_1 \\ C & D_2 & 0 \end{array}\right]$$

Duality

The *dual* of system $\Sigma : (A, B, C, D)$ is defined as $\Sigma^{\mathsf{T}} : (A^{\mathsf{T}}, C^{\mathsf{T}}, B^{\mathsf{T}}, D^{\mathsf{T}})$.



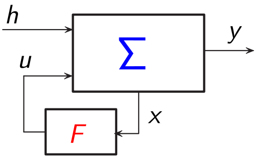
Consider the interconnection of systems shown above: the overall dual system is obtained by reversing the order of serially connected systems and interchanging branching points with summing junctions and vice versa:

$$\Sigma = \left[\begin{array}{ccc|c} A_1 & 0 & 0 & B_1 \\ 0 & A_2 & 0 & B_2 \\ B_{3,1}C_1 & B_{3,2}C_2 & A_3 & 0 \\ \hline 0 & 0 & C_3 & 0 \end{array}\right], \qquad \Sigma^{\mathsf{T}} = \left[\begin{array}{ccc|c} A_1^{\mathsf{T}} & 0 & C_1^{\mathsf{T}}B_{3,1}^{\mathsf{T}} & 0 \\ 0 & A_2^{\mathsf{T}} & C_2^{\mathsf{T}}B_{3,2}^{\mathsf{T}} & 0 \\ 0 & 0 & A_3^{\mathsf{T}} & C_3^{\mathsf{T}} \\ \hline B_1^{\mathsf{T}} & B_2^{\mathsf{T}} & 0 & 0 \end{array}\right]$$

Outline

- The beginning
- The main tools
- **The most important solved problems**

The disturbance decoupling problem (1969-70)



The disturbance decoupling problem (1969)

Equations

$$\begin{aligned} \dot{x}(t) &= A\,x(t) + B\,u(t) + H\,h(t) \\ y(t) &= C\,x(t) \end{aligned}$$

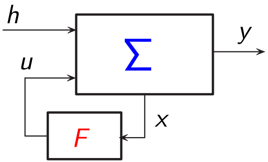
Let $\mathcal{B} = \mathrm{im}\,B$, $\mathcal{H} = \mathrm{im}\,H$, $\mathcal{C} = \ker C$.

It will be shown that the solution in geometric terms depends on A, $\mathcal{B}$, $\mathcal{H}$, $\mathcal{C}$.

W. M. Wonham and A. S. Morse, "Decoupling and pole assignment in linear multivariable systems: a geometric approach," *SIAM J. Contr.*, vol. 8, no. 1, pp. 1–18, 1970.

G. Basile and G. Marro, "L'invarianza rispetto ai disturbi studiata nello spazio degli stati," in *Rendiconti della LXX Riunione Annuale AEI*, paper 1.4.01, Rimini, Italy, 1969,

The disturbance decoupling problem (1969-70)

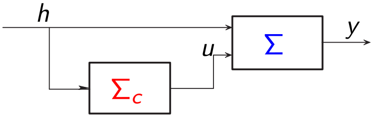


The disturbance decoupling problem

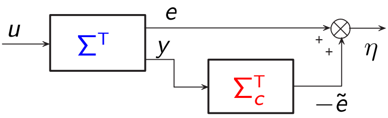
The solution of a synthesis problem with geometric techniques as a rule includes

- A structural condition
- A stabilizability condition

Other basic problems (primal and dual)



Measurable signal decoupling

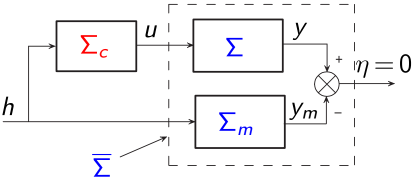


Unknown-input observation

S. P. Bhattacharyya, “Disturbance rejection in linear systems,” *Int. J. Systems Science*, vol. 5, no. 7, pp. 931–943, 1974.

R. Laschi and G. Marro, “Alcune considerazioni sull’osservabilità dei sistemi dinamici con ingressi inaccessibili” in *Rendiconti della LXX Riunione Annuale AEI*, paper 1.1.06, Rimini, Italy, 1969,

A review of exact feedforward model following

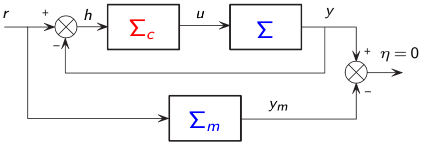


Exact model following as a measurable disturbance decoupling problem with stability.

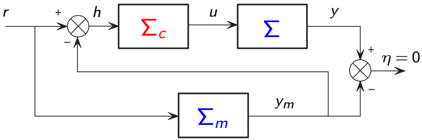
NOTE:

If Σ_m consists of p independent SISO systems connected in parallel, we achieve exact row-by-row decoupling *at no cost*.

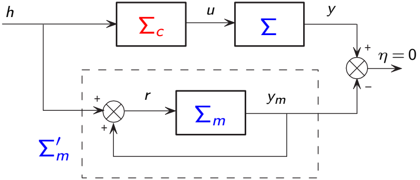
From exact feedforward to exact feedback model following



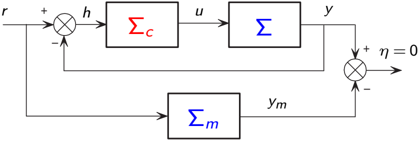
Exact feedback model following.



A structurally equivalent connection.

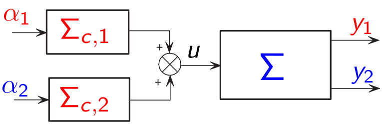


Another structurally equivalent connection.

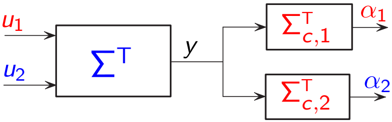


Exact feedback model following, possibly with row-by-row decoupling and multiple internal models.

Other basic problems (primal and dual)



The noninteracting control problem



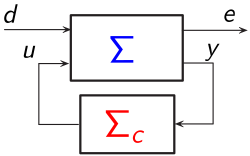
The fault detection and isolation problem

A. S. Morse and W. M. Wonham, "Decoupling and pole assignment by dynamic compensation," *SIAM J. Contr.*, vol. 8, no. 3, pp. 317–337, 1970.

G. Basile and G. Marro, "A state space approach to noninteracting controls," *Ricerche di Automatica*, vol. 1, pp. 68–77, 1970.

M. Massoumnia, G. C. Verghese, and A. S. Willsky, "Failure detection and identification," *IEEE Trans. Aut. Contr.*, vol. 34, pp. 316–321, 1989.

Other basic problems



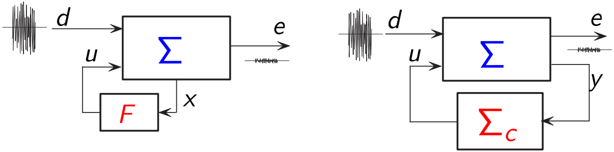
Disturbance Decouplig Problem with dynamic output feedback

J. M. Schumacher, "Compensator synthesis using (C,A,B)-pairs," *IEEE Trans. Aut. Contr.*, vol. AC-25, pp. 1133–1138, 1980.

J. C. Willems and C. Commault, "Disturbance decoupling with measurement feedback with stability or pole placement," *SIAM J. Contr. Optim.*, vol. 19, pp. 490–504, 1981.

G. Basile, G. Marro, and A. Piazzi, "Stability without eigenspaces in the geometric approach: Some new results," in *Frequency Domain and State Space Methods for Linear Systems*, C. A. Byrnes and A. Lindquist, Eds., pp. 441–450. North-Holland (Elsevier), Amsterdam, 1986.

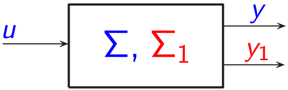
From exact to H_2-optimal and H_∞-optimal



The H_2-optimal disturbance decoupling problem

- The main criticism to which the geometric approach is subjected to is that exact solutions to the above problems cannot be implemented in practice because of uncertainties in parameters.
- In this approach standard optimal control and filtering problems can be treated in a minimal H_2 or H_∞ norm context by simply substituting system Σ with the corresponding Hamiltonian system.
- Thus the geometric approach provides insight and tools for the treatment of singular and cheap cases of optimal control.
- Interesting results in this area are due to Stoorvogel, Chen, Saberi and Sannuti (1990-2000).

Spectral factorization and H_2-optimal model following



The Stoorvogel problem

$$\dot{x}(t) = A\,x(t) + B\,u(t)$$
$$y(t) = C\,x(t) + D\,u(t)$$

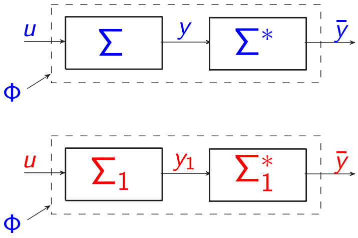
$$y_1(t) = C_1\,x(t) + D_1\,u(t)$$

A. Stoorvogel, A. Saberi, and B. M. Chen, "Full and reduced observer-based controller design for H_2 optimization," *Int.. J. Contr.*, vol. 58, pp. 803–834, 1993.

A. Stoorvogel, "The singular H_2 control problem," *Automatica*, vol. 28, pp. 627–631, 1992.


```
>> sys=ss(A,B,C,D);
>> [C1,D1]=stoor(A,B,C,D,[Tc]);
```

Spectral factorization and H_2-optimal model following



The spectral factorization problem
(the spectrum Φ is the same)

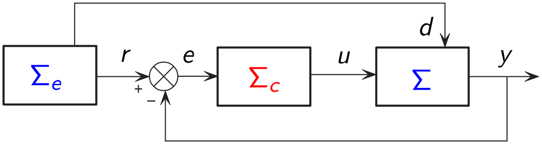
```
>> sys=ss(A,B,C,D);
>> adjsys=ss(-A',-C',B',D');
```

(adjoint system)

```
>> sys1=ss(A,B,C1,D1);
>> adjsys1=ss(-A',-C1',B',D1');
```

G. Marro, F. Morbidi and D. Prattichizzo, "A geometric solution to the cheap spectral factorization problem", in *Proc. ECC2009*, Budapest, 2009.

The multivariable regulator problem

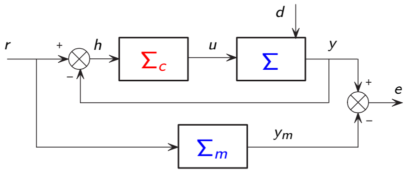


The multivariable regulator problem with internal model (1977-1987)

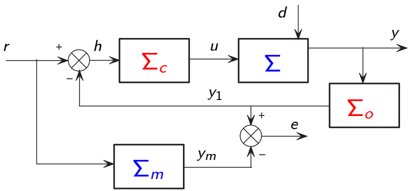
B. A. Francis, "The linear multivariable regulator problem," *SIAM J. Contr. Optim.*, vol. 15, pp. 486–505, 1977.

G. Basile, G. Marro and A Piazzi, "Revisiting the regulator problem in the geometric approach. Part II: Asymptotic tracking and regulation in the presence of disturbances," J. Optim. Theory Appl., 53, 23–26, 1987.

The multivariable regulator problem



Exact feedback model following (2002)



H_2-optimal feedb. model following (2007)

G. Marro, F. Morbidi and D. Prattichizzo, "H_2 pseudo-optimal model following: a geometric approach," Proc. 3rd IFAC Symposium on System Structure and Control, Foz de Iguaçu, Brazil, October 17-19, 2007.

Conclusion

- The geometric approach has been developed for about forty years gradually, as new linear and nonlinear problems arose and were analyzed.

- It provides a significant insight into systems and control, based on very few elementary tools, on which the overall theory is based.

- Although the geometric tools are very simple and supported by exhaustive computational machinery, it is rather difficult to get a complete panorama of them, since their presentation in the literature by several authors is not uniform in style and has very often been covered in unnecessarily heavy mathematics.

A great friend

To conclude, let me remember a great friend, Giuseppe Basile, who worked with me on the above research and is no longer with us.

I recall, in particular, his intuition for new ideas and his lively enthusiasm in discussion for developing them.

Thank you!